Topology

Back Paper Examination

1st January 2013

Instructions: All questions carry equal marks. All sets and collections in the questions are assumed to be non-empty!

1. Describe all distinct topologies that can be put on a set of size 3.

2. Let X and Y be connected spaces and A and B be proper subsets of X and Y respectively. Prove that $(X \times Y) \setminus (A \times B)$ is connected.

3. Prove that there is no surjective continuous map from the closed interval $[0, 1]$ to the real line $\mathbb{R}$. What can you say about the open interval $(0, 1)$?

4. Show that every locally compact Hausdorff space is regular.

5. Show that a connected metric space having more than one point is uncountable.

6. Assume that X has a countable basis and let A be an uncountable subset of X. Prove that uncountably many points of A are limit points of A.

7. Give an example to show that a Hausdorff space with a countable basis need not be metrizable.

8. Let X and Y be two spaces. Show that the projection map $\pi : X \times Y \to X$ is a covering map if and only if Y has discrete topology.

9. **(Path lifting lemma)** Let $p : E \to B$ be a covering map with $p(e_0) = b_0$. Prove that any path $f : [0, 1] \to B$ with $f(0) = b_0$ has a unique lifting to a path $\overline{f} : [0, 1] \to E$ with $\overline{f}(0) = e_0$.

10. If X is path connected and if x and y are two points of X, prove that $\pi_1(X, x)$ and $\pi_1(X, y)$ are isomorphic groups.